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TITLE SIMULATING QCD WITH DYNAMICAL WILSON AND STAGGERED FERMIONS

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Simulating QCD with dynamical Wilson and staggered fermions [§]

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ABSTRACT

I present a summary of the work done during the last year by the Los Alamos lattice collaboration (C. Baillie, R. Gupta, G. Kilcup, G. Guralnik, A. Patel and S. Sharpe) to study QCD with the Hybrid Monte Carlo Algorithm. The specific questions we have addressed are the finite temperature transition for both staggered and Wilson fermions and screening in the heavy $q\bar{q}$ potential with Wilson fermions. I also present a discussion of the efficiency of the fermion update algorithm.

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1. Introduction

In this talk I will discuss the following five topics:

1) The finite temperature transition for 4 flavors of staggered fermions with emphasis on finite size effects.
2) Tests of the Hybrid Monte Carlo Algorithm (HMCA) at small quark masses for staggered fermions.
3) Evaluation of HMCA for Wilson fermions and some first results for the finite temperature transition.
4) Screening in the heavy $q\bar{q}$ potential in QCD for $N_f = 2$ Wilson fermions.
5) Comparison of HMCA and Langevin algorithms

All the calculations were done using the HMCA [1] and to a large extent are studies of the algorithm. The particular variation of the algorithm we use and its implementation is discussed in [2] [3] and I direct readers to these for background. Also, the motivation and physical significance of the calculations has been discussed before so in this talk I will concentrate on new results, technical details and future prospects.

2. The finite temperature transition at $m_q = 0.1$

This is a progress report on an ongoing study of the finite temperature transition with 4 flavors of staggered fermions. Previous calculations show that on 4×6^3 lattices, the transition is first order for all quark masses [4] . The data on 4×10^4, 4×12^3 and 4×14^3 lattices at intermediate values of the quark mass $0.2 - 0.5$ does not show a clear two state signal, so the order is still a subject of debate [5] . In this talk I present evidence for a two state signal (first order transition) at $m_q = 0.1$ on up to 4×10^3 lattices. New results, the time histories for $\langle uloop \rangle$, $\langle uline \rangle$, $\langle \overline{\chi}\chi \rangle$ on 4×10^3 lattices, are shown in figure 1. The histograms of the data for $\langle uline \rangle$ at β_c are shown in figure 2 and those for $\langle \overline{\chi}\chi \rangle$ in figure 3. By comparing the data obtained on different lattice sizes we conclude that large finite volume corrections are present. The histograms for 4×10^3 lattice at $\beta = 5.1325$ still show a double peak structure but the two peaks overlap. From the tunnellings observed in the time histories and from comparitive runs at $\beta = 5.13$ and 5.135 we conclude that the data still shows a two state signal.

For large volumes the expected behavior at a first order transition is that the peaks get narrower and more clearly resolved as the spatial volume is increased. Finite size scaling at first order transitions predicts that finite volume corrections to the discontinuity in an order parameter vanish as $1/volume$. While our data is consistent with this behavior, the rate of decrease is large enough to encompass the case that there is no discontinuity in the thermodynamic limit. Therefore we need to run on larger lattices to settle whether the transition survives the infinite volume limit for $m_q = 0.1$.

The up shot of the calculations done over the last year is that we need to undertake a detailed and careful finite volume analysis in order to ascertain the order of the transition. Since there are no clear general statements about the order (like a first order transition for all values of m_q), it is important to do the finite volume study for the case of interest: two light u and d quarks and one intermediate mass s quark. Also, this calculation should be done at weaker coupling where scaling holds. Such a calculation is not possible with HMCA (only multiples of 4 staggered flavors are allowed). It is also unrealistic to try to extract hard numbers at strong coupling due to large flavor symmetry breaking effects. These limitations are over and above the question of approximate algorithms. In the end the first quantity that we will extract with good precision is the transition temperature since the location of the transition is the easiest to measure. Meanwhile, we turn to a technical exercise: does the strong first order transition seen at small quark mass ($m_q \leq 0.05$) with 4 staggered flavors survive the infinite volume limit. This study is also motivated by our desire to understand the performance of HMCA at small quark mass.

There is still some lingering debate over whether one gets the same results using different algorithms. The answer is obscured by the fact that different calculations use different parameters, scan the transition region in different ways and because the approximate algorithms have errors. For example the critical value of $6/g^2$ is shifted towards lower values of $6/g^2$ in runs using approximate algorithms. The magnitude of the shift depends on the algorithm and the step size, and ranges from ≈ 0.01 to 0.05, with the hybrid algorithm having the smallest error. Keeping this caveat in mind, I believe that we are seeing the same gross features in all simulations.

To facilitate comparison, let me end this section by stating what we are doing that is different from other groups: (1) we use an exact algorithm as does the FSU group [6] and our results are in agreement. (2) We reduce the error in $\langle \overline{\psi}\psi \rangle$ by averaging over 25 pseudofermion fields. Note that in the noisy fermion implementation of the Langevin and hybrid algorithms, one only gets an estimator for $\langle \overline{\psi}\psi \rangle$ on a given background gauge configuration. To the best of our knowledge no other group has used this variance reduction trick. We consider this important because for $m_q \geq 0.1$, the fluctuations in the estimator $\langle \overline{\psi}\psi \rangle$ measured with a single pseudofermion field are larger than the discontinuity.

3. Tests of HMCA at small quark masses:

In [3] we presented analytic arguments that showed that asymptotically the performance of HMCA will slow down as $V^{5/4}$. The extra power of 1/4 comes in because the step size ϵ has to be decreased in order to maintain a constant acceptance. In addition, the algorithm has four other sources of critical slowing down. (1) For fixed β and acceptance rate, ϵ has to be decreased with m_q; (2) there is an increase in the number of iterations of a linear solver, say conjugate gradient algorithm, necessary to calculate $(M^\dagger M)^{-1}\phi$ as

a function of m_q and volume; (3) the decorrelation time increases as $m_q \to 0$; and (4) the usual critical slowing down as $g \to 0$. In this talk I give estimates for these and some of the numerical data supporting the case.

(1) The step-size ϵ has to be decreased as $m_q \to 0$. This is because the error in ΔH integrated over a molecular dynamics trajectory depends on m_q due to contributions to $\dot{P}$ from the fermionic part of the action. The fermion action $S_F = \phi^\dagger (M^\dagger M)^{-1} \phi$ is independent of m_q since we update ϕ using the gaussian noise method. The leading term in the step-size error in H is given by the third power of the derivative of S_F with respect to the link variables U (see eqn. 5.5 in ref [3] for details). Since each derivative of S_F gives a power of $1/m$, the total change in H integrated over a trajectory of length $1/\epsilon$ goes as ϵ^2/m^3. Thus ϵ has to be decreased as $m_q^{3/2}$ in the limit $m_q \to 0$. This is consistent with our data: on 4×6^3 lattices we had to use $\epsilon = 0.06$, 0.026 and 0.011 at $m_q = 0.1$, 0.05 and 0.025 respectively in order to maintain $\approx 70\%$ acceptance. In these runs we carried out tests with different precision to remove any extra factors that may require a decrease of ϵ due to residual errors in the calculation $(M^\dagger M)^{-1}\phi$ at any stage of the evolution.

(2)The number of conjugate gradient iterations increases as $1/m_q$, *i.e.* they are dictated by the lowest eigenvalue of the Dirac matrix.

(3) On decreasing m_q, we estimate that the increase in de-correlation time grows like ξ^2 where $\xi = 1/m_\pi$. This gives another factor of $1/m_q$ in computer time. Putting all these factors together HMCA degrades as $V^{5/4}/m_q^{3.5}$.

(4) The final source of slowing down that occurs as $g \to 0$ is due to the freezing of the gauge fields. We have not done tests at different values of the $6/g^2$ and I give a hand-waving conjecture based on the previous statements. The simulation time per trajectory at two values of $6/g^2$ that have the same lattice volume and pion correlation length should only differ by a prefactor. However, note that as the coupling gets weaker, a fixed pion correlation length corresponds to a heavier pion due to a change in the lattice scale a. Furthermore, in terms of decorrelated trajectories, one expects at least the same growth in auto-correlataions as for the pure gauge theory.

To conclude, even though HMCA combines the best features of Langevin and molecular dynamics algorithms, it is no panacea. Major improvements can be made by devising algorithms for linear solvers that overcome the $1/m_q$ slowing down in addition to reducing the prefactor, and by finding a more efficient pre-processor than molecular dynamics (none is known at present). The present best iterative algorithm for inverting the staggered fermion matrix is straight-forward conjugate gradient and for Wilson fermions it is ILU or preconditioned minimal residue with overrelaxation [7] . Unfortunately, our tests show that these only reduce the prefactor. Of the new algorithms with potential, fourier acceleration does not give any significant improvement at these couplings and lattice sizes and multigrid is not yet a working scheme for QCD.

To evaluate the performace of HMCA at small quark masses we again decided to study the finite temperature transition. We find that to sit on top of the transition (observe flip flops) requires careful tuning. Even though β_{md} is a free parameter as far as the final distribution is concerned, it needs to be carefully tuned to observe phase transitions especially when the transition is sharp and when data from a finite run is analysed. Emperically we find that one should try to tune β_{md} to the value at which the transition occurs for the uncorrected hybrid algorithm and then tune β to get equal population in the two phases. Since this not a well defined process we had to make a number of runs in close vicinity of the transition. We find that at $m_q = 0.025$, the transition is sharp and consequently the nature of the flip-flops changes on changing either β value by as little as 0.002.

Our data for the time histories at the transition for $m_q = 0.05$ and 0.025 are shown in figure 4 and 5. Note that the transition occurs at larger β than with approximate algorithms -- a feature now observed in all studies.

The study of the finite temperature transition at small masses also allowed us to address another technical question about HMCA: to what accuracy does one run the inversion in order that there is no bias in the configurations generated? In the molecular dynamics evolution steps, this question is relevant only for the case when we use a smart guess for $\chi \equiv (M^\dagger M)^{-1}\phi$ rather than doing a fixed number of iterations starting from a constant initial guess for χ. We give two tests: (1) For the HMCA, the unbiased Boltzmann distribution satisfies the identity first pointed out by Creutz [8]

$$\langle e^{\Delta H} \rangle = 1 \quad . \tag{3.1}$$

We find that for low convergence criterion this identity is visibly violated. It is therefore a very useful test and should be monitored during the run. (2) In the accept/reject step, ΔH should lie mostly in the interval $[-1,1]$ independent of the lattice volume in order to maintain $\approx 70\%$ acceptance. This final number is a result of cancellation between three numbers of $O(10^5)$ for a 16^4 lattice. By running a few trajectories with different convergence criterion one can quickly decide what is safe since ΔH is once again very sensitive to the accuracy. This test also raises the issue of machine precision: for 32 bit machines, it will be necessary to break up the global sums (dot products of vectors over the whole lattice) into sums over time slices and then accumulate these. In the linear solver, information propagates locally and this involves operations between numbers that are all of the same order of magnitude. So one does not *a priori* expect that 64 bit precision is necessary. However, to really decide whether the solution converges to a desired precision with 32 bit words we need more tests at small quark mass.

4. Finite temperature transition with Wilson fermions

In order to calculate the hadronic properties from lattice QCD using a fast and reliable

algorithm that incorporates the effects of dynamical Wilson fermions, one needs to first determine

a) For a given coupling $\beta \equiv 6/g^2$ the critical value of the hopping parameter, κ_c, at which the pion mass vanishes.

b) for a given $6/g^2$ and κ (which specifies the lattice spacing a), the position of the finite temperature transition *i.e.* $N_t^c = 1/(T_c a)$ where T_c is the transition temperature in physical units. Note that N_t^c as defined above need not be an integer.

These two calculations provide 1) a rough estimate of the scale (through T_c), 2) determine, based on experience with pure gauge simulations, what lattice size should be used to avoid severe finite size effects and 3) indicate the interesting physical region where the u and d quark masses are small and vacuum polarization effects should be discernable.

I estimate that with current algorithms and available computer power the range of $6/g^2$ that can be explored in the next few years using small dynamical quark masses is $6/g^2 \in [5.3, 5.6]$. This range corresponds to pure gauge simulations carried out in the range $6/g^2 \sim [5.6, 6.1]$. Present results of pure gauge simulations show that one starts to make contact with continuum physics for $6/g^2 > 6.0$. Thus a crude extrapolation from pure gauge results suggests that it will be possible to simulate QCD with light dynamical fermions close to the scaling region in the next few years. Here, I present our first results towards this goal with 2 flavors of Wilson fermions.

We chose $6/g^2 = 5.3$ for our first runs because we wanted to test the algorithm at small quark masses and to get a glimpse of the effects of dynamical fermions. For this we had to work at strong coupling to avoid finite size/temperature effects and poor statistics due to large auto-correlations. From a hadron spectrum analysis we estimate that the chiral limit for $6/g^2 = 5.3$ is at $\kappa_c = 0.169$ [9] . Our first goal is to locate the transition for $N_t = 4$ and 6. In figure 6, I show the time history at $\kappa = 0.1575$, the transition point for $N_t = 4$. For comparison, the histograms for $\langle u\ell m e \rangle$ at $\kappa = 0.157$, 0.1575 and 0.158 are shown in figure 7. The hysteresis study for $N_t = 6$ is not yet complete and our preliminary estimate of the transition is $\kappa \approx 0.167$. This value of κ corresponds to roughly the strange quark mass for the zero temperature theory. The bad news is that at this mass (which is still rather heavy) the algorithm is already severely affected by the small eigenvalues of the Dirac operator and configurations evolve very slowly.

The one big surprise from this study is that the $N_t = 4$ transition for 2 Wilson flavors occurs at much heavier quark mass than for the staggered case. The Wilson mass $m_q = log(1 + 0.5(1/\kappa - 1/\kappa_c))$ for $\kappa = 0.1575$ with $\kappa_c = 0.169$ is ≈ 0.22. An estimate for the location of the transition for the 2 flavor staggered case is $m_q a \approx 0.03$ for $N_t = 4$ and $\beta = 5.3$. I have extracted this number from the study by Gottlieb *et al.* after making some allowance for the expected shift in β_c on going from hybrid to HMCA simulations. If, on the other hand, we determine the equivalent staggered mass by equating the hadron spectrum

then $m_q a \approx 0.1$. These two estimates highlight a subtlety: in hadron mass calculations the predominant effect of quark loops is to dress the valence quark propagator. This is exemplified by the successful use of the phenomerological mass formulas involving the constituent quark mass instead of the current mass. In the confined phase, chiral symmetry is spontaneously broken for both pure gauge theory and full QCD and low energy gluons provide the dominant contribution to the dressing; consequently, the hadron spectrum is not vastly affected by the sea quarks. The remaining difference (0.1 versus 0.22) in the estimate of the quark mass is analogous to that seen in quenched calculations of the hadron spectrum between the two types of fermions [10] .

The important lesson here is the difference in the matching staggered mass as estimated from three observables; (1) the chiral transition, (2) Wilson loops and (3) the hadron spectrum. This is an indication of very large lattice effects between the two kinds of fermions and also due to the flavor symmetry breaking for staggered fermions at strong coupling. Note that it is mainly in the calculation of the determinant (or it to some power) that the flavor symmetry violation produces the largest effect and the resultant configurations correspond to a heavier effective sea quark mass. These features should be kept in mind in analyzing the contribution of sea quarks.

5. Screening in the heavy $q\bar{q}$ potential

Confinement in pure gauge QCD is visualized as the formation of a color electric flux-tube between two isolated static quarks. The strength of this linear confinement is characterized by the string tension, and a simple phenomenological spin-independent potential is taken to have the form $\alpha/r + \sigma r$. This picture changes when dynamical quarks of mass m are included. It becomes favorable for the string to break due to the creation of a $q\bar{q}$ pair from the vacuum when the energy in the string exceeds $2M$, where M is the constituent mass (except when the mesons formed are pions). The resulting configuration is a pair of mesons interacting through a screened potential. Thus the shape of the potential one extracts from lattice QCD depends on the dynamical mass m. For very heavy quarks, one expects little deviation from the pure gauge potential for separation $r < M/(2\sigma)$ except for a renormalization of the scale. For physical u and d quark mass, one expects the linear rise in the potential to cease at the length scale of confinement $\approx 0.5 - 1 fermi$. The goal of lattice simulations is to map out the potential so that we can predict the toponium spectra.

We performed simulations on a 8^4 lattice at six values of $\kappa = 0.156, 0.158, 0.160, 0.162, 0.165, 0.167$. We restricted our calculations to $6/g^2 = 5.3$ in order to measure zero temperature values even at the lowest quark mass. For $\kappa > 0.165$ we find that the convergence of matrix inversion is severly affected by small modes in the Dirac operator and the auto correlation time becomes large (we could not measure it reliably but it could

be as large as a few hundred trajectories). Thus a better algorithm and substantially more computer time is required to probe the smaller quark mass region.

The Wilson loop expectation values are given in table 1. Since we can barely measure 4×4 loops it is not possible to extract a potential reliably. Therefore, we first study a simpler problem: the behavior of Wilson loops as a function of the quark mass in order to expose the screening effect of dynamical quarks. To do this we match loop expectation values against the pure gauge data and analyse the shift of the pure gauge coupling, $\Delta\beta$, as a function of the loop size and the quark mass.

Theoretically, we expect that in addition to an overall renormalization of the bare coupling $6/g^2$, the presence of dynamical quarks change the area law for large Wilson loops, found in pure gauge theories, to a perimeter law. Thus, in the matching process the effect of vacuum polarization should become discernable at some loop size which depends on the quark mass. For loops with an area larger than this size we expect to find that $\Delta\beta \rightarrow \infty$ as $Area \rightarrow \infty$ because loops measured in presence of dynamical quarks are not supressed by the area term. This effect should become more pronounced with decreasing quark mass and should start occuring at smaller loop area.

$< Wilson\ Loop >$ with 2 flavors						
kappa	0.1560	0.1580	0.1600	0.1620	0.1650	0.1670
nconf	900	1340	1020	1350	1710	2370
1x1	0.4847(3)	0.4909(2)	0.4957(2)	0.5016(4)	0.5152(6)	0.5299(6)
1x2	0.2472(3)	0.2548(3)	0.2606(3)	0.2679(5)	0.2852(5)	0.3047(4)
1x3	0.1274(3)	0.1337(2)	0.1386(2)	0.1451(4)	0.1604(4)	0.1785(4)
1x4	0.0658(2)	0.0703(2)	0.0740(2)	0.0788(3)	0.0905(3)	0.1050(4)
2x2	0.0712(2)	0.0770(2)	0.0812(2)	0.0870(4)	0.1014(4)	0.1195(4)
2x3	0.0214(1)	0.0245(1)	0.0267(1)	0.0299(2)	0.0385(3)	0.0507(3)
2x4	0.0065(1)	0.0079(1)	0.0089(1)	0.0105(2)	0.0149(2)	0.0219(2)
3x3	0.0040(1)	0.0051(1)	0.0058(1)	0.0070(1)	0.0105(1)	0.0169(2)
3x4	0.0007(1)	0.0011(1)	0.0013(1)	0.0016(1)	0.0031(1)	0.0060(1)
4x4	0.0000(1)	0.0001(1)	0.0002(1)	0.0004(1)	0.0007(1)	0.0020(1)

Table 1: *Wilson loop expectation values for for two flavors of Wilson fermions as a function of the loop size and κ.*

In table 2 we show the shift in the pure gauge coupling required to match a given Wilson loop expectation value as a function of loop area and for the six values of κ. The shift in 1×1 loops is much larger than that predicted by the lowest order hopping parameter expansion *i.e.* $48\kappa^4$. Also, the $\Delta\beta$ increases as a function of the loop's area and as the

$\Delta\beta$ from linear interpolation						
kappa	0.1560	0.1580	0.1600	0.1620	0.1650	0.1670
1x1	0.155(1)	0.177(1)	0.196(1)	0.218(2)	0.266(2)	0.320(2)
1x2	0.160(1)	0.184(1)	0.203(1)	0.226(2)	0.277(1)	0.335(1)
1x3	0.162(1)	0.187(1)	0.206(1)	0.229(2)	0.282(1)	0.341(1)
1x4	0.162(1)	0.188(1)	0.208(1)	0.231(2)	0.284(1)	0.344(2)
2x2	0.165(1)	0.192(1)	0.212(1)	0.236(2)	0.289(1)	0.353(2)
2x3	0.166(1)	0.197(1)	0.216(1)	0.240(2)	0.295(2)	0.363(2)
2x4	0.165(2)	0.200(2)	0.218(2)	0.243(2)	0.298(2)	0.368(2)
3x3	0.171(2)	0.202(2)	0.222(2)	0.246(3)	0.301(2)	0.375(2)
3x4	0.167(8)	0.209(7)	0.224(4)	0.243(7)	0.306(4)	0.381(3)
4x4	-	-	0.209(27)	0.266(13)	0.308(13)	0.401(6)

Table 2: *The shift in pure gauge β necessary to match Wilson loops with two flavors of sea quarks as a function of the loop size and κ.*

quark mass is decreased. This is shown in figure 8 where we plot $\Delta\beta - \Delta\beta(1 \times 1)$ versus loop area for the six values of κ. No error bars are plotted since the data is given in table 2. The effect is barely significant for $\kappa = 0.156 - 0.162$ and clear only at $\kappa = 0.167$ *i.e.* at or lighter than the strange quark system. To conclude, our data only qualitatively shows the expected screening behavior.

Previous studies of screening can be divided into two categories: Fukugita *et al.* do not see a marked difference in Wilson loop expectation value compared to pure gauge results other than an overall shift in $6/g^2$ (expected behavior for heavy quarks) while de Forcrand *et al.* [11] and Grady *et al.* [12] do. Unfortunately, some of the results of de Forcrand *et al.* (3 degenerate Wilson flavors) are in the high temperature phase where the expected screening is qualitatively different. The results of Grady *et al.* are for 4 staggered flavors and we believe that the reason that in their data the potential does not show any mass dependence is because their effective quark mass is very large due to staggered flavor symmetry breaking. Our results do show the expected screening behavior as a function of the quark mass. To quantify the effects of dynamical fermions on the $q\bar{q}$ potential we have to measure larger loops at smaller m_q and g.

The above results have implications for other physical observables like the hadron spectrum. If these are extracted from simulations done at $\kappa < 0.167$ then there is little hope of doing much better than the quenched approximation since the effect of the sea quarks will be small.

6. Efficiency of HMCA versus Langevin

We present a numerical comparison of the efficiency of the HMCA and Langevin algorithms using Wilson loops as probes. For the HMCA we measured up to 3×3 loops at $6/g^2 = 5.5$ and $\kappa = 0.15$. For these parameter values we do extract zero temperature results even on a 6^4 lattice since the finite temperature transition for $N_t = 1/T_c a = 6$ is at $\kappa \approx 0.1565$. The results using Langevin update are given by Fukugita *et al.* [13] for three values of the step size, $\delta\tau = 0.02, 0.01, 0.005$. They also provided the $\delta\tau = 0$ values using a linear extrapolation of their data. The complete data are given in table 3. One finds good agreement between the two calculations and for algorithm comparison we assume that the error estimates are reliable and equal to first approximation. Therefore, to estimate the CPU time for the two calculations we simply add up the total number of time steps. Note that each time step requires a matrix inversion which accounts for $\approx 90\%$ of the total CPU time. This inversion can be carried out using the same optimal algorithm in both cases.

The HMCA calculation used 80 trajectories of length 40 (with a step size $\epsilon = 0.04$) for a total of 3200 steps. The results of Langevin simulation show very large step size errors, *i.e.* the 3×3 loop expectation value changes by over 100% between $\delta\tau = 0.02$ and 0.0 (this is one reason why Langevin evolution is not a good preprocessor for HMCA). Therefore, we shall assume that Langevin simulations need to be done at at least three values of $\delta\tau$. Fukugita *et al.* evaluate expectation values over 20,30,20 configurations with $\delta\tau = 0.02, 0.01, 0.005$ respectively for a total of 8000 steps (20*50 + 30*100 + 20*200). The two simulations used lattice volumes different by a factor of ≈ 4 (6×8^3 for HMCA versus $9^3 \times 18$ for Langevin). Assuming that CPU time grows like $V^{5/4}$, the HMCA will require ≈ 6 times as long on a $9^3 \times 18$ lattice. This factor of six will be partly offset by the gain in statistics due to the larger volume and the absence of thermalization overhead of multiple $\delta\tau$ runs.

Loop	$\delta\tau = 0.02$	$\delta\tau = 0.01$	$\delta\tau = 0.005$	$\rightarrow \delta\tau = 0.0$	HMCA
1×1	0.5137(9)	0.5257(7)	0.5330(9)	0.5389(10)	0.5384(10)
1×2	0.2799(9)	0.2958(7)	0.3053(9)	0.3131(13)	0.3125(15)
1×3	0.1548(8)	0.1689(7)	0.1779(8)	0.1848(11)	0.1845(15)
2×2	0.0947(9)	0.1080(9)	0.1165(12)	0.1228(12)	0.1227(16)
2×3	0.0339(5)	0.0419(5)	0.0474(7)	0.0511(8)	0.0516(11)
3×3	0.0084(6)	0.0120(5)	0.0144(7)	0.0161(6)	0.0164(5)

Table 3: *Comparison of the Wilson loop data with 2 flavors of Dynamical Wilson fermions at $\kappa = 0.15$ and $\beta = 5.5$ The first three columns give results obtained by [13]using the Langevin algorithm while the forth is the linear extrapolation of their data to $\delta\tau = 0$. The fifth column gives our results using the HMCA algoriuthm.*

The above crude analysis shows that for gluonic observables HMCA is at least as efficient as Langevin simulations. For calculations involving external quark propagators (as in hadron spectrum and matrix elements calculations) HMCA gains in storage and CPU since fewer configurations need to be analysed as there is no extrapolation $\delta\tau \to 0$ needed. Finally, the fact that in Langevin simulations a linear extrapolation to zero step size is still an approximation further tips the scale in favor of HMCA.

One of the criticisms leveled at approximate algorithms is whether they produce configurations that are in the same universality class as QCD. The fact that we find agreement with results from an exact algorithm when a simple linear extrapolation in $\delta\tau$ is used is encouraging. However, we need an unequivocal answer (numerical "proof" will be hard to establish). If universality holds for small $\delta\tau$, then we can forget about the $\delta\tau \to 0$ extrapolation; instead we should focus on showing that mass-ratios become constant as $g \to 0$ and $m_q \to 0$. I look forward to the next lattice meeting in anticipation of an answer to this important question.

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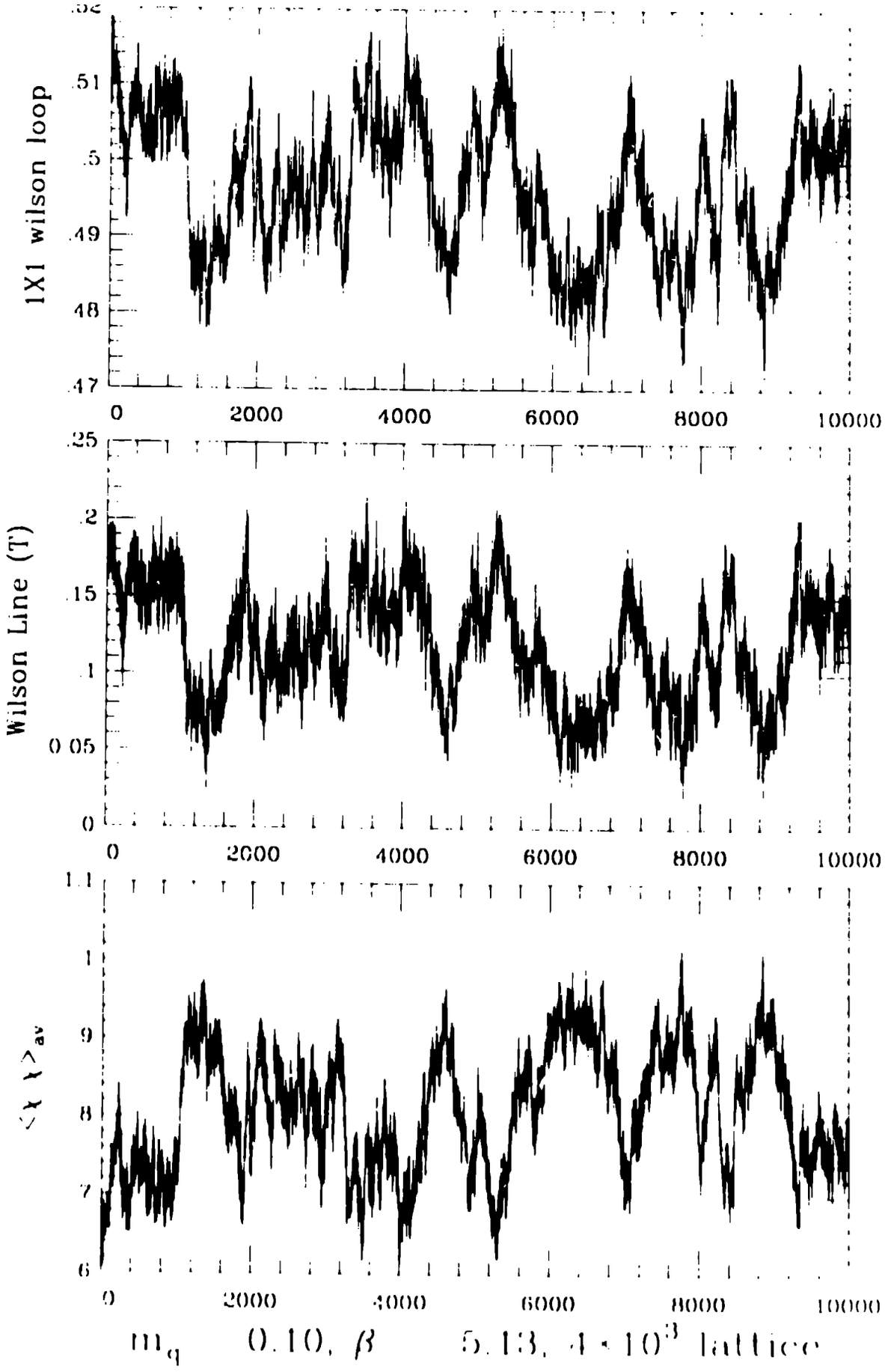

1X1 wilson loop
.52
.51
.5
.49
.48
.47
0
2000
4000
6000
8000
10000
Wilson Line (T)
.25
.2
.15
.1
0 05
0
0
2000
4000
6000
8000
10000
$\langle \chi \chi \rangle_{av}$
1.1
1
9
8
7
6
0
2000
4000
6000
8000
10000
m_q 0.10, β 5.13, 4×10^3 lattice

FIG 1

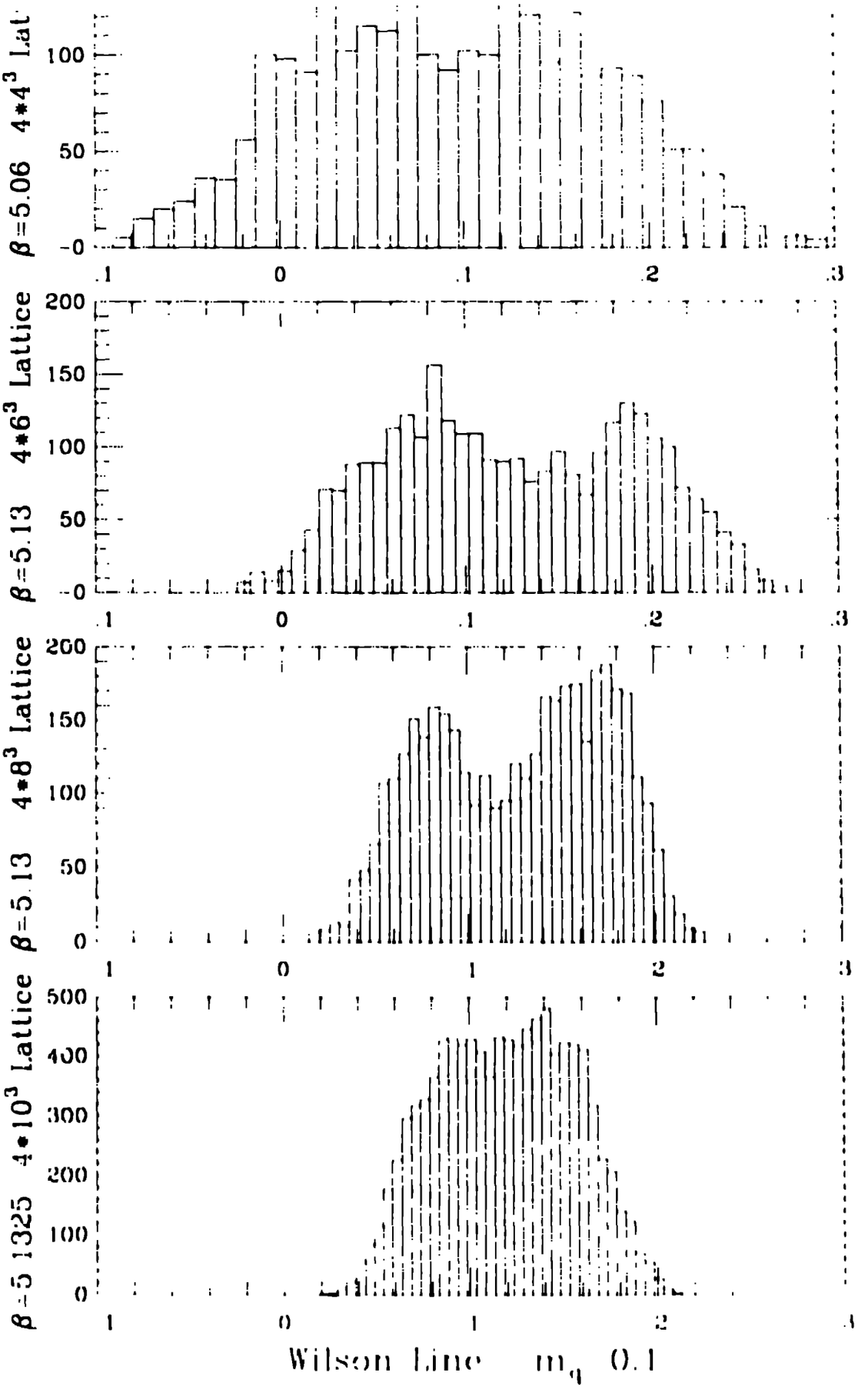

β=5.06 4*4³ Lat
β=5.13 4*6³ Lattice
β=5.13 4*8³ Lattice
β=5 1325 4*10³ Lattice
Wilson Line m_q 0.1

Fig 2

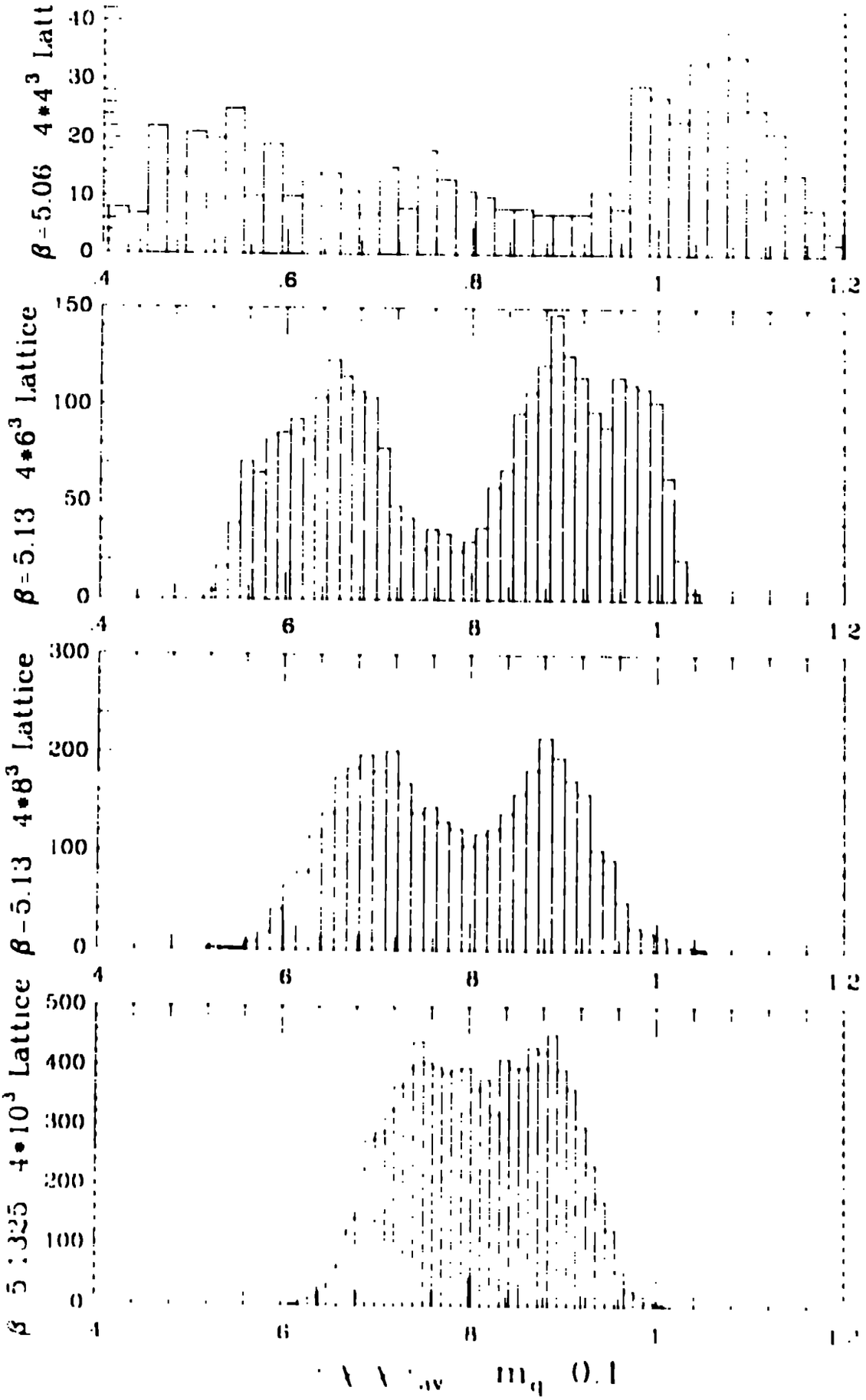
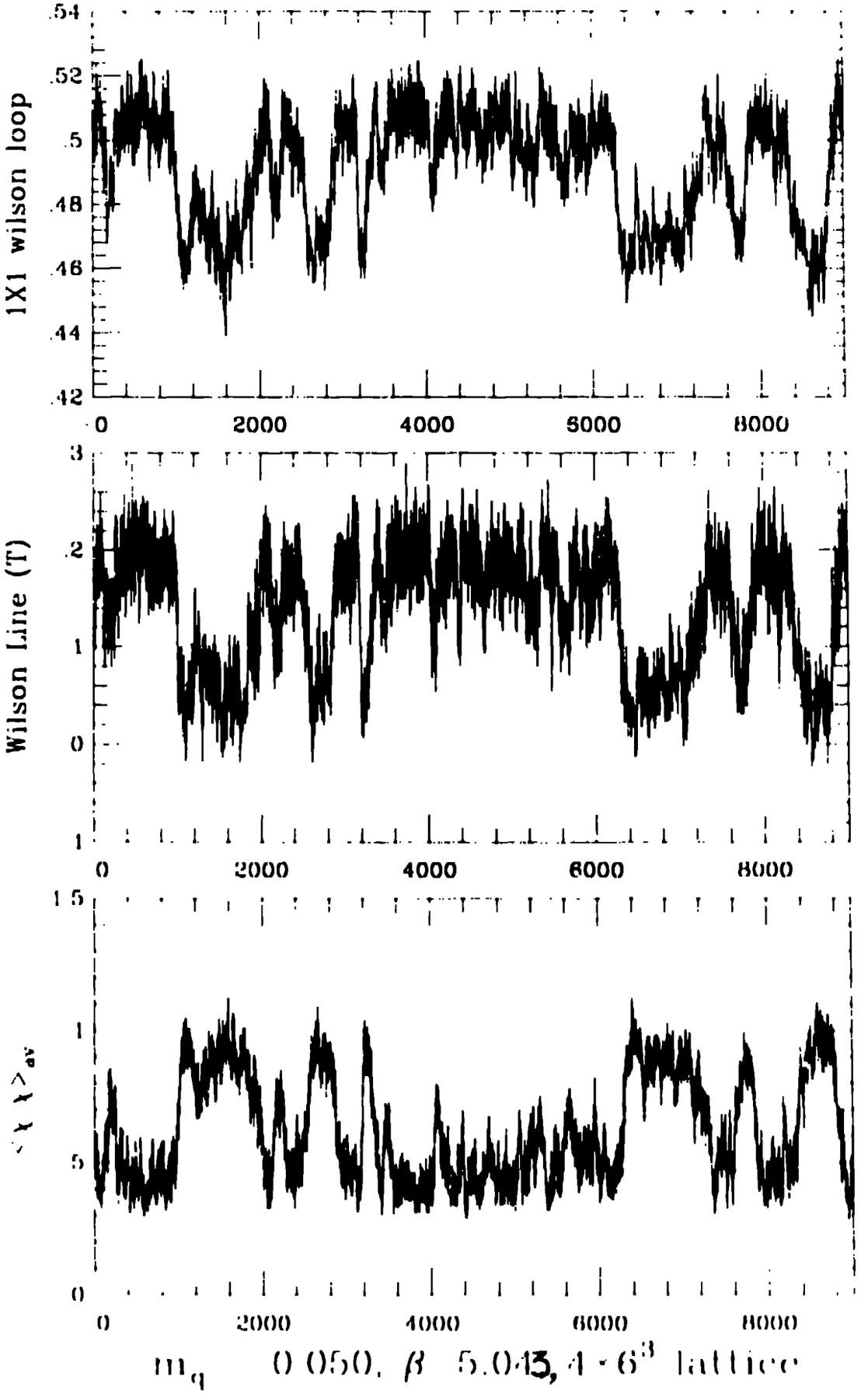
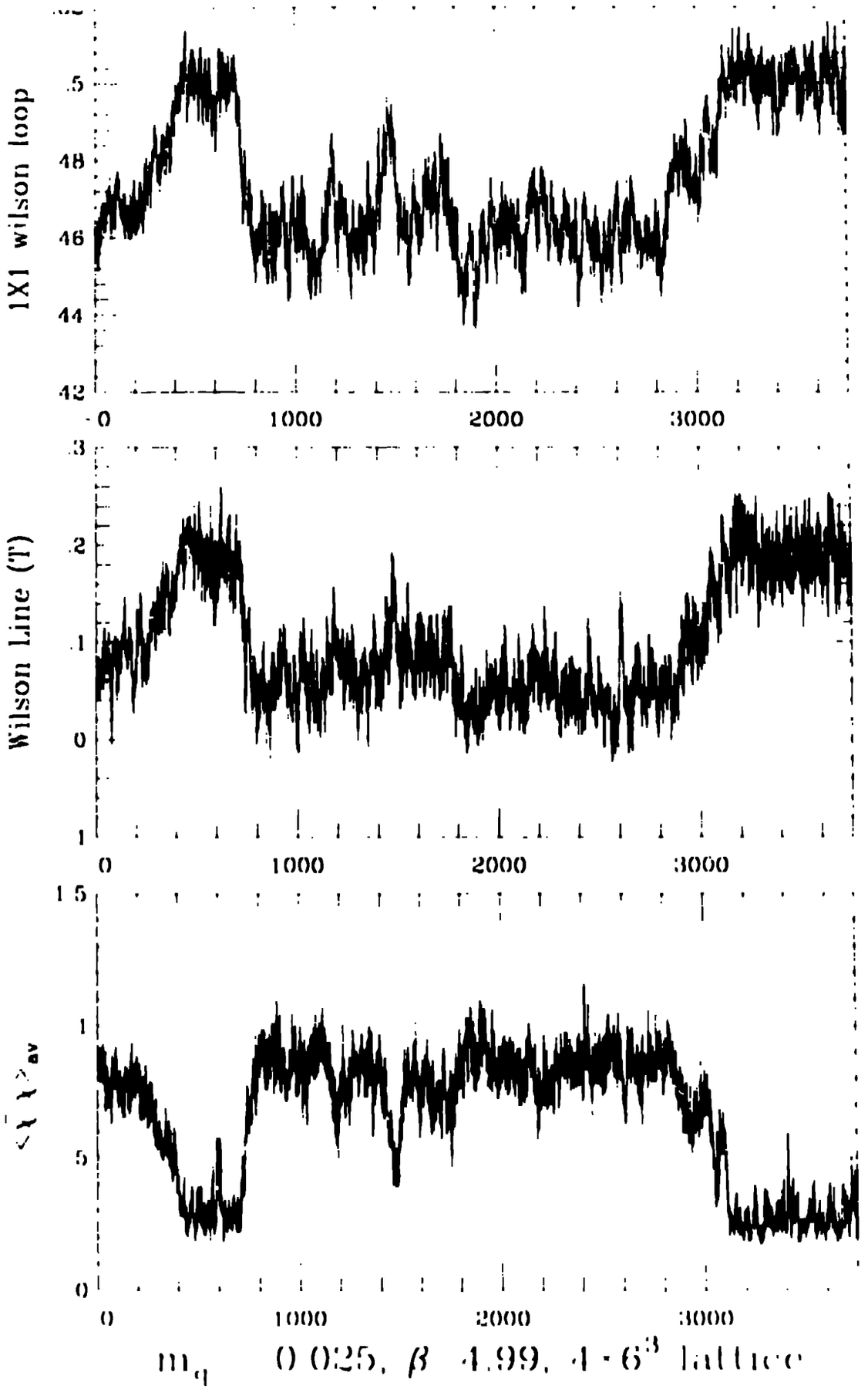

β=5.06 4*4^3 Latt
β=5.13 4*6^3 Lattice
β=5.13 4*8^3 Lattice
β 5.325 4*10^3 Lattice
m_q 0.1

FIG. 3



m_q 0.050, β 5.043, $4 \cdot 6^3$ lattice

Fig 4



m_q 0.025, β 4.99, $4 \cdot 6^3$ lattice

Fig 5

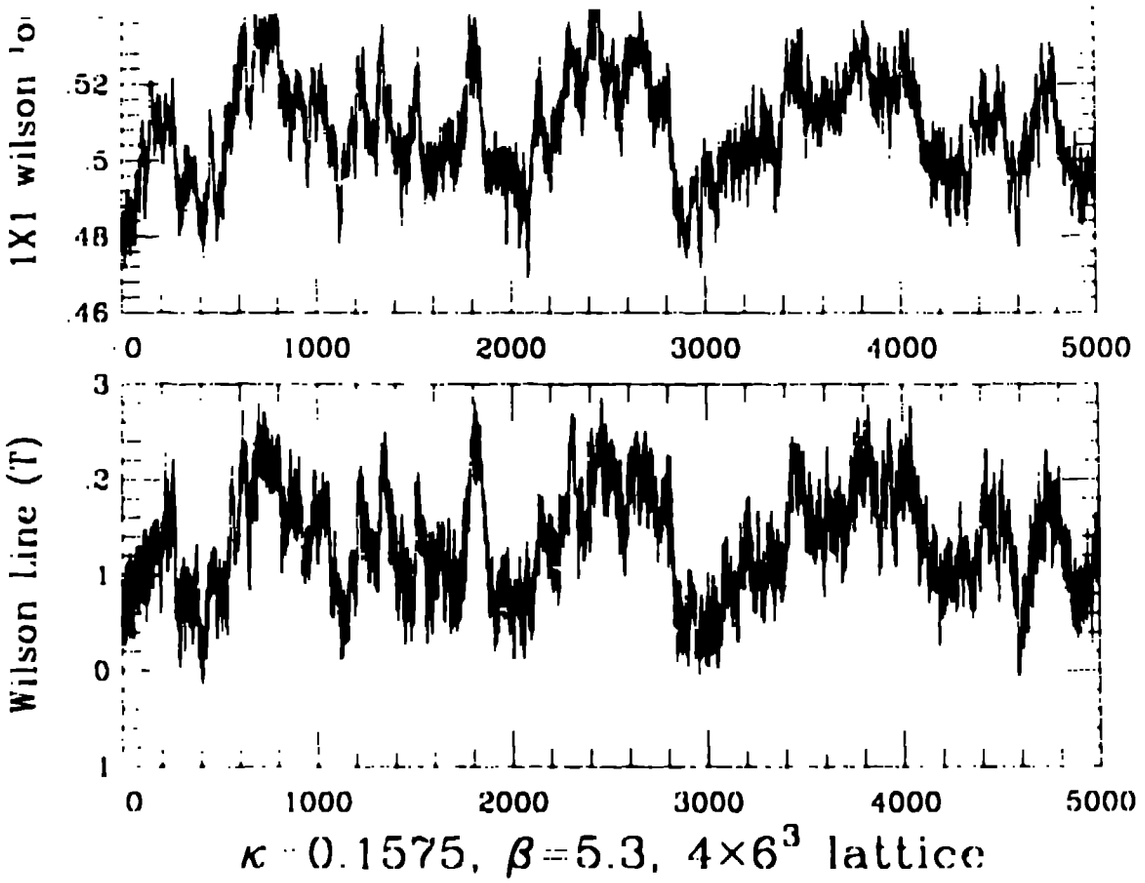

1X1 wilson lo
.52
.5
.48
.46
0
1000
2000
3000
4000
5000
Wilson Line (T)
3
2
1
0
1
0
1000
2000
3000
4000
5000
κ=0.1575, β=5.3, 4×6³ lattice

FIG 6

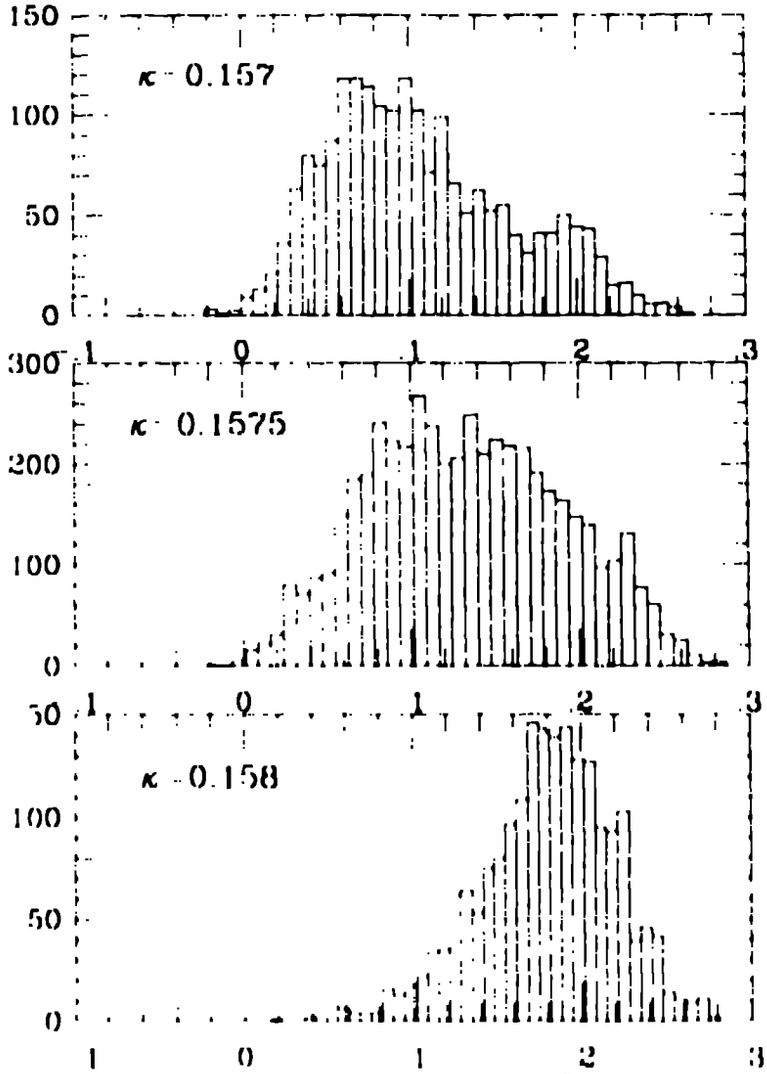


Fig 7 Histogram of Wilson line on $4\ 6^3$ lattices at β 5 3

Fig 7

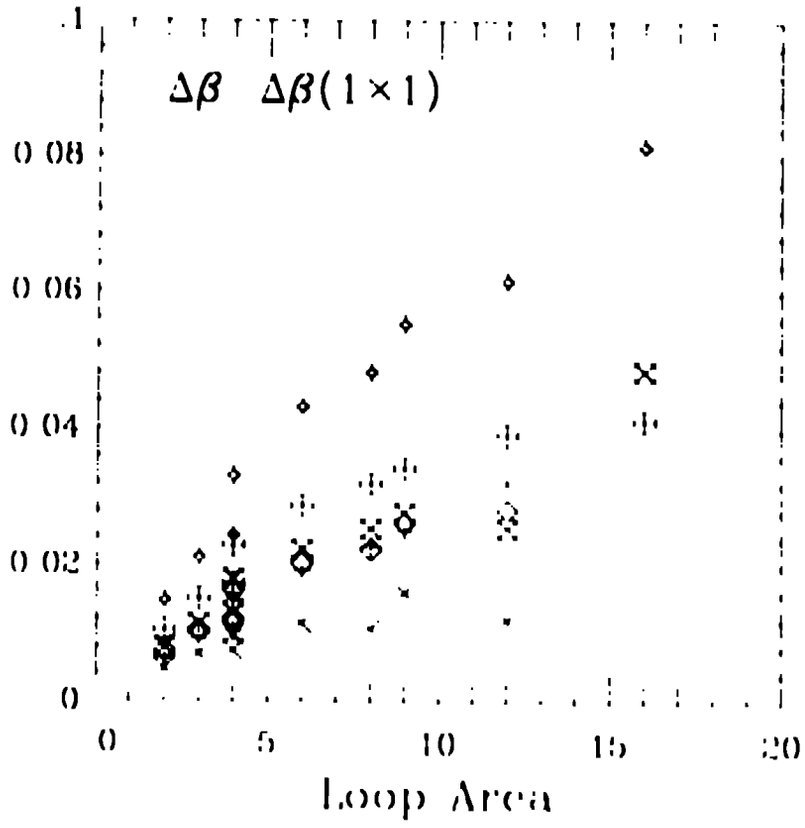


Fig. 5